Bayesian Inference in Factor Models

Introduction

- Macro researchers usually have dozens or hundreds of time series variables to work with
- This has led to large VARs
- Bayesian priors used to surmount challenge of over-fitting
- Priors are on parameters
- Instead of using prior shrinkage on parametrers, why not compress/shrink the data itself
- Work with smaller, more parsimonious model, with compressed data
- This is the idea motivating factor models
- These are state space models so Bayesian inference straightforward

The Static Factor Model

- y_t is $M \times 1$ vector of time series variables
- M is very large
- y_{it} denote a particular variable.
- Simplest static factor model:

$$y_t = \lambda_0 + \lambda f_t + \varepsilon_t$$

- f_t is $q \times 1$ vector of unobserved latent factors (where $q << M$)
- Factors contain information extracted from all the M variables.
- Same f_t occurs in every equation for y_{it} for $i = 1, ..., M$
- But different coefficients (λ is an $M \times q$ matrix of so-called factor loadings).

- Note that restrictions are necessary to identify the model
- Common to say ε_t is i.i.d. $N(0, D)$ where D is diagonal matrix.
- Implication: ε_{it} is pure random shock specific to variable i, co-movements in the different variables in y_t arise only from the factors.
- Note also that $\lambda f_t = \lambda CC^{-1} f_t$ which shows we need identification restriction for factors too.
- Different models arise from different treatment of factors.
- Simplest is: $f_t \sim N(0, I)$
- This can be interpreted as a state equation for "states" f_t
- Factor models are state space models — so our MCMC tools of for state space methods can be used.

The Dynamic Factor Model (DFM)

- In macroeconomics, usually need to extend static factor model to allow for the dynamic properties which characterize macroeconomic variables.
- A typical DFM:

$$
\begin{aligned}
y_{it} &= \lambda_{0i} + \lambda_i f_t + \varepsilon_{it} \\
f_t &= \Phi_1 f_{t-1} + .. + \Phi_p f_{t-p} + \varepsilon_t^f
\end{aligned}
$$

- f_t is as for static model
- λ_i is $1 \times q$ vector of factor loadings.
- Each equation has its own intercept, λ_{0i}.
- ε_{it} is i.i.d. $N\left(0, \sigma_i^2\right)$
- f_t is VAR with ε_t^f being i.i.d. $N\left(0, \Sigma^f\right)$
- Note: usually ε_{it} is autocorrelated (easy extension, omitted here for simplicity)

Replacing Factors by Estimates: Principal Components

- Proper Bayesian analysis of the DFM treats f_t as vector of unobserved latent variables.
- Before doing this, we note a simple approximation.
- The DFM has similar structure to regression model:

$$y_{it} = \lambda_{0i} + \widetilde{\lambda}_{0i} f_t + .. + \widetilde{\lambda}_{pi} f_{t-p} + \widetilde{\varepsilon}_{it}$$

- If f_t were known we could use Bayesian methods for the multivariate Normal regression model to estimate or forecast with the DFM.
- Principal components methods to can be used to approximate f_t.
- Precise details of how principal components is done provided many places

Treating Factors as Unobserved Latent Variables

- DFM is a Normal linear state space model so use Bayesian MCMC methods for state space models
- A bit more detail on MCMC algorithm:
- Conditional on the model's parameters, $\Sigma^f, \Phi_1, .., \Phi_p, \lambda_{0i}, \lambda_i, \sigma_i^2$ for $i = 1, .., M$, use (e.g.) Carter and Kohn algorithm to draw f_t
- Conditional on the factors, measurement equations are just M Normal linear regression models.
- Since ε_{it} is independent of ε_{it} for $i \neq j$, posteriors for $\lambda_{0i}, \lambda_i, \sigma_i^2$ in the M equations are independent over i
- Hence, the parameters for each equation can be drawn one at a time (conditional on factors).
- Finally, conditional on the factors, the state equation is a VAR
- Any of the priors for Bayesian VARs discussed before can be used.

The Factor Augmented VAR (FAVAR)

- DFMs are good for forecasting (extract all information in huge number of variables)
- VARs are good for macroeconomic policy (e.g. impulse responses).
- Why not combine DFMs and VARs together to get model which can do both?
- FAVAR results
- Bernanke, Boivin and Eliasz (2005, QJE) is pioneering paper

- FAVAR modifies DFM by adding other explanatory variables:

$$y_{it} = \lambda_{0i} + \lambda_i f_t + \gamma_i r_t + \varepsilon_{it}$$

- r_t is $k_r \times 1$ vector of observed variables of key interest.
- E.g. Bernanke, Boivin and Eliasz (2005) set r_t to be the Fed Funds rate (monetary policy instrument)
- All other assumptions are same as for the DFM.
- Note: by treating r_t in this way, we can isolate a "monetary policy shock" and calculate impulse responses

- FAVAR state equation extends DFM state equation to include r_t:

$$\begin{pmatrix} f_t \\ r_t \end{pmatrix} = \widetilde{\Phi}_1 \begin{pmatrix} f_{t-1} \\ r_{t-1} \end{pmatrix} + .. + \widetilde{\Phi}_p \begin{pmatrix} f_{t-p} \\ r_{t-p} \end{pmatrix} + \widetilde{\varepsilon}_t^f$$

- where all assumptions are same as DFM with extension that $\widetilde{\varepsilon}_t^f$ is i.i.d. $N\left(0, \widetilde{\Sigma}^f\right)$
- MCMC is very similar to that for the DFM and will not be described here.
- Similar ideas:
- Normal linear state space algorithms can draw f_t
- Measurement equation is series of regressions (conditional on factors)
- The state equation is a VAR (conditional of factors)

The TVP-FAVAR

- With VARs: began with constant parameter model
- then we said it is good to allow the VAR coefficients to vary over time: homoskedastic TVP-VAR
- then we said good to allow for multivariate stochastic volatility: heteroskedastic TVP-VAR
- Can do the same with FAVARs
- Note: just as with TVP-VARs, TVP-FAVARs can be over-parameterized and careful incorporation of prior information or the imposing of restrictions (e.g. only allowing some parameters to vary over time) can be important in obtaining sensible results.

- A TVP-FAVAR is just like a FAVAR but with t subscripts on parameters:

$$y_{it} = \lambda_{0it} + \lambda_{it} f_t + \gamma_{it} r_t + \varepsilon_{it},$$

- $$\begin{pmatrix} f_t \\ r_t \end{pmatrix} = \widetilde{\Phi}_{1t} \begin{pmatrix} f_{t-1} \\ r_{t-1} \end{pmatrix} + .. + \widetilde{\Phi}_{pt} \begin{pmatrix} f_{t-p} \\ r_{t-p} \end{pmatrix} + \widetilde{\varepsilon}_t^f$$

- All each ε_{it} to follow univariate stochastic volatility process
- $var\left(\widetilde{\varepsilon}_t^f\right) = \widetilde{\Sigma}_t^f$ has multivariate stochastic volatility process of the form used in Primiceri (2005).
- Finally, the coefficients (for $i = 1, .., M$) $\lambda_{0it}, \lambda_{it}, \gamma_{it}, \widetilde{\Phi}_{1t}, .., \widetilde{\Phi}_{pt}$ are allowed to evolve according to random walks (i.e. state equations of the same form as in the TVP-VAR complete the model).
- All other assumptions are the same as for the FAVAR.

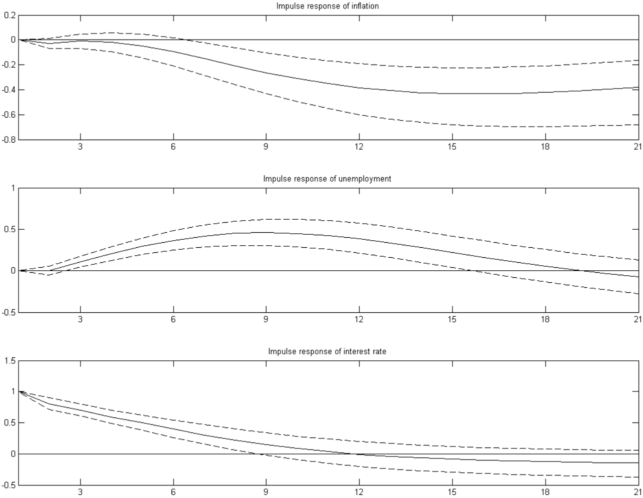
Bayesian Inference in the TVP-FAVAR

- I will not provide details of MCMC algorithm
- Note only it adds more blocks to the MCMC algorithm for the FAVAR.
- These blocks are all of forms discussed in previous lectures.
- E.g. error variances in measurement equations drawn using the univariate stochastic volatility algorithm of Kim, Shephard and Chib (1998).
- Multivariate stochastic volatility algorithm of Primiceri (2005) can be used to draw $\widetilde{\Sigma}_t^f$.
- The coefficients $\lambda_{0it}, \lambda_{it}, \gamma_{it}, \widetilde{\Phi}_{1t}, .., \widetilde{\Phi}_{pt}$ are all drawn using algorithm for Normal linear state space model

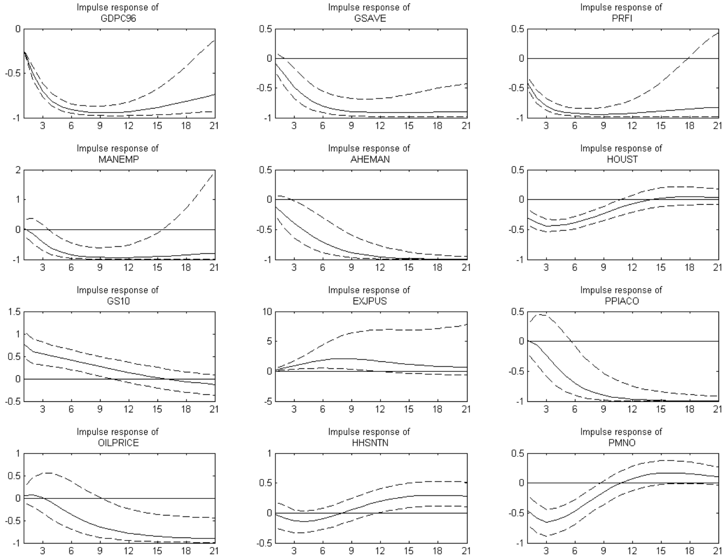
Empirical Illustration of the FAVAR and TVP-FAVAR

- 115 quarterly US macroeconomic variables spanning 1959Q1 though 2006Q3.
- Transform all variables to be stationarity.
- What variables to put in r_t?
- Inflation, unemployment and the interest rate.
- FAVAR is same as VAR from previous illustrations, but augmented with factors, f_t
- We use 2 factors and 2 lags in state equation
- Identify impulse responses to a monetary policy shock
- Solid lines are posterior medians
- Dashed lines in some of following figures denote credible intervals

Impulse Responses of Main Variables

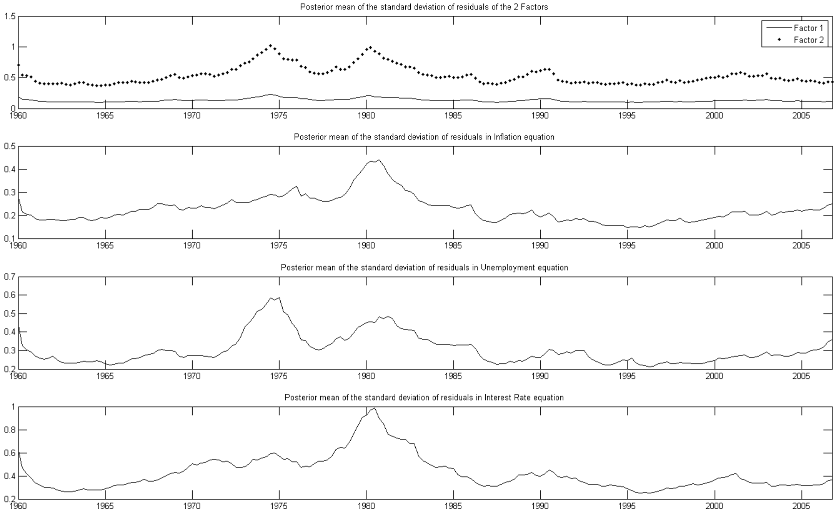


Impulse Responses of Selected Other Variables

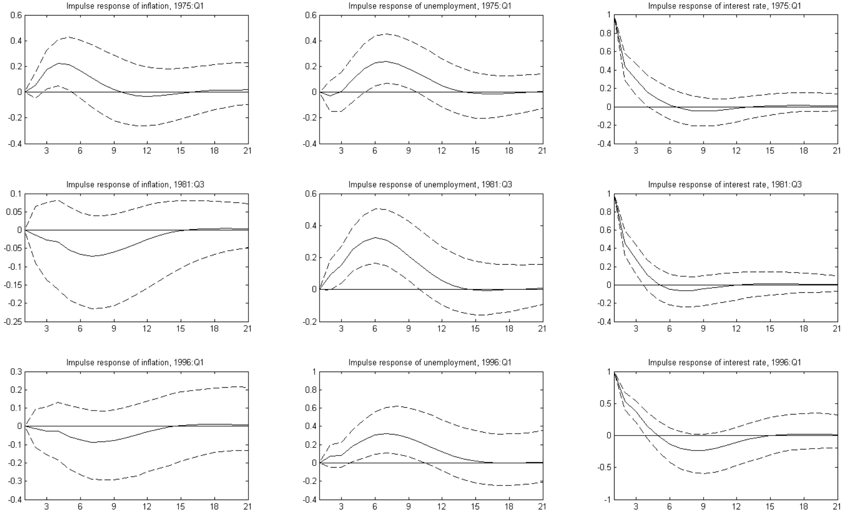


- Now TVP-FAVAR
- Illustrate time varying volatility of equations for r_t and factor equations
- Impulse responses at three different time periods

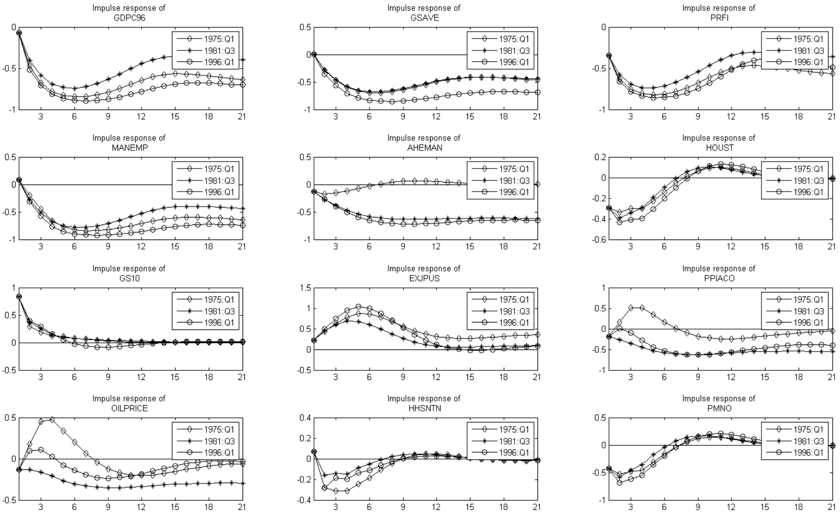
Time Varying Volatilties in Some Key Equations



Impulse Responses of Main Variables to Monetary Policy Shock



Impulse Responses of Selected Other Variables



Summary

- Factor methods are an attractive way of modelling when the number of variables is large
- DFMs often are good for forecasting
- FAVARs good for macroeconomic policy (e.g. to do impulse response analysis)
- Recently TVP versions of these models have been developed
- Bayesian inference in TVP-FAVAR puts together MCMC algorithm involving blocks from several simple and familiar algorithms.